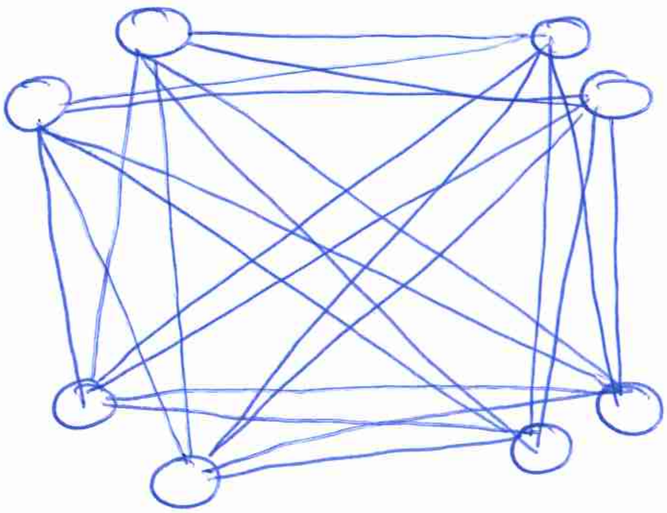
① Turán Graph



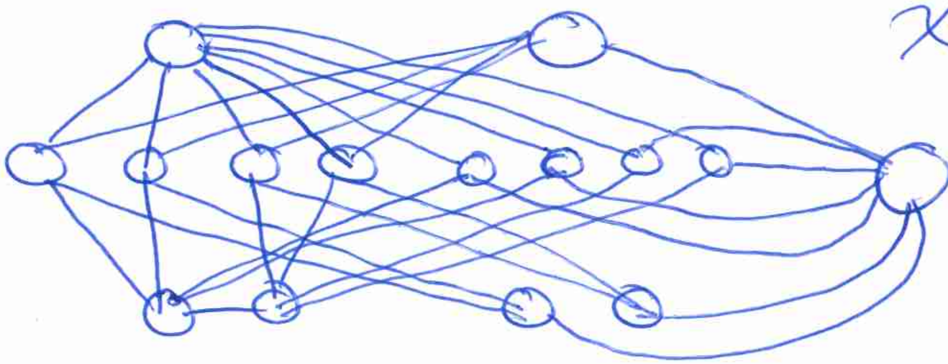
$\chi(G) = 4$

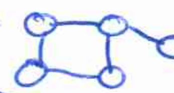
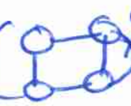
$|V(G)| = 8$

$|E(G)| = 24$

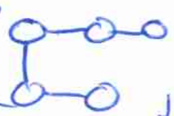
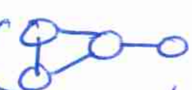
② Mycielski's construction

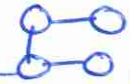
$\chi(G') = 4$



③ $\chi($  $, k) = \chi($  $, k) - \chi($  $, k)$

can remove

$= \chi($  $, k) - \chi($  $, k)$

$- \chi($  $, k) + \chi($  $, k)$

$= k(k-1)^{(5-1)} - \chi($  $, k) + \cancel{\chi}($  $, k)$

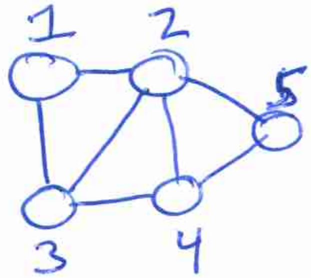
$- k(k-1)^{(4-1)} + k(k-1)(k-2)$

$= k(k-1)^4 - k(k-1)^3 + k(k-1)^2 - k(k-1)^3$

$+ k(k-1)(k-2)$

(4) Content won't be on final

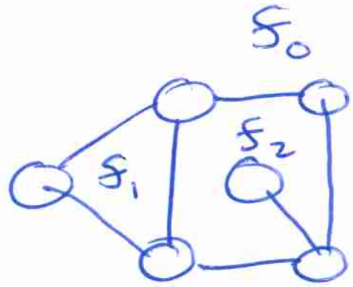
(5)

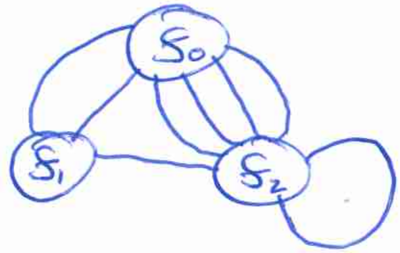


Perfect = Yes

Perfect $\Leftrightarrow$ $\exists$ simplicial elimination ordering

(6)





$n - e + f = 2$

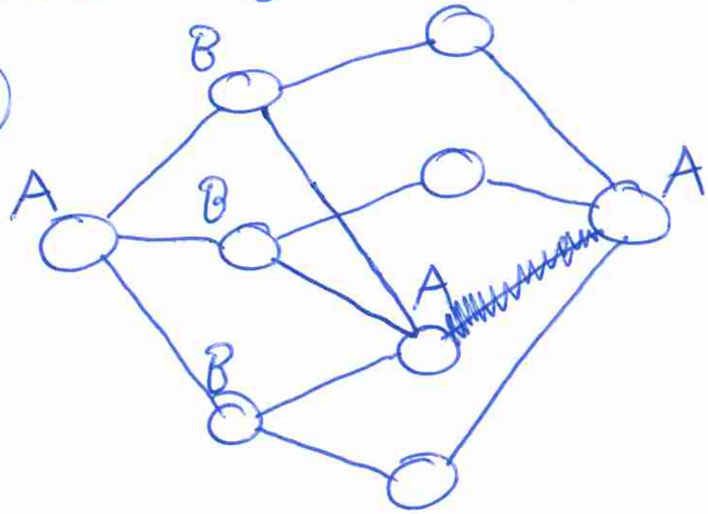
$6 - 7 + 3 = 2$ ✓

(7) Because $|E(G)| < 10$ it cannot contain K_5 or a K_5 K.S.

Because $|E(G)| = 9$ and doesn't contain $K_{3,3}$ it also can't contain a $K_{3,3}$ K.S. (would need ≥ 10 edges)

No K_5 or $K_{3,3}$ K.S. $\Leftrightarrow$ planar

(8)



Note: sets of A and B together form the "hubs" of a $K_{3,3}$ K.S.

$\Rightarrow$ not planar

(9) G has no claws or OOTs
$\Rightarrow \exists H : G = L(H)$

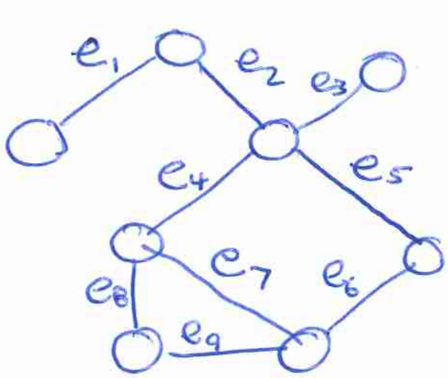
- We can solve the minimum edge coloring problem on H in $O(n^k)$
- We can calculate H from G in $O(n)$
- We can transfer the solution from edges of H to vertices of G in $O(n)$

$$\text{Total} = \underbrace{O(n^k)}_{=\text{poly}} + \underbrace{O(n) + O(n)}_{\text{linear}}$$

$= \text{poly} + \text{linear} = \text{polynomial time}$

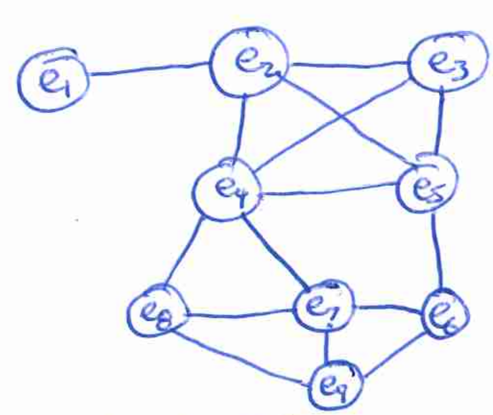
(10) We didn't end up covering this in class. But if the connectivity of G is greater than the independence number, G has a Hamiltonian cycle. $K(G) = 5 > \alpha(G) = 4$

(11)



G

$\Rightarrow$



$L(G)$

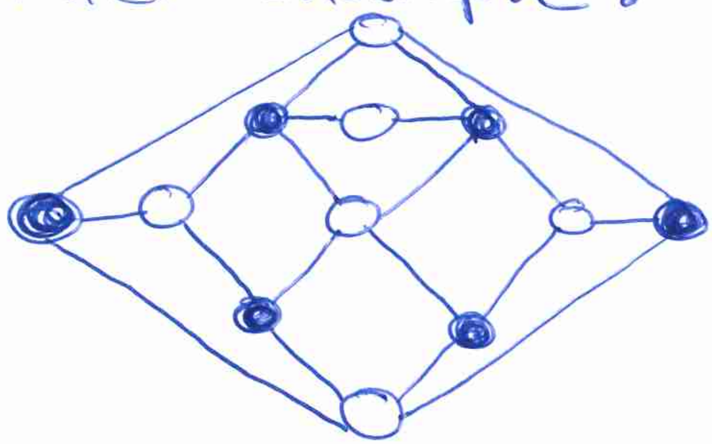
(12) Note: $\forall S \subseteq V(G): c(G-S) \leq |S|$

$|X| = |Y|$ if bipartite

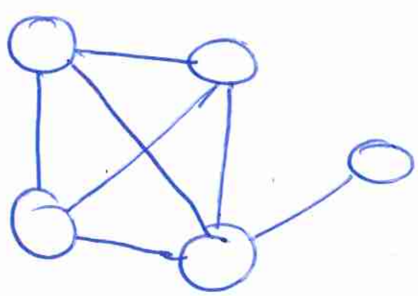
2-connectedness

are all <u>necessory</u> but not <u>sufficient</u> conditions for Hamiltonianess. ← not a real word

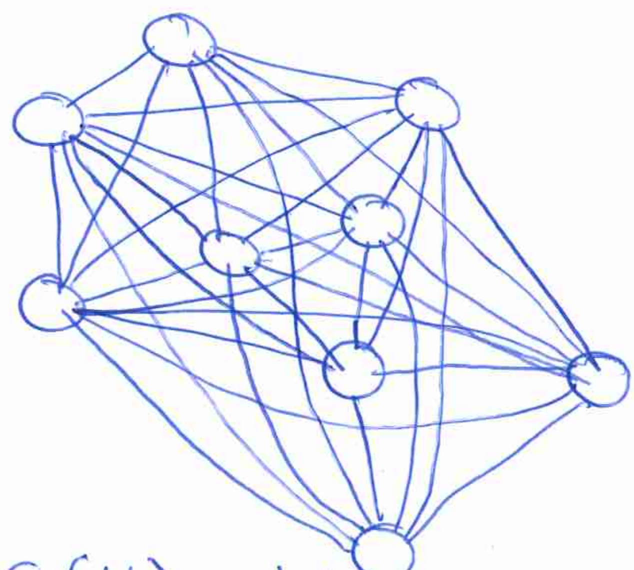
counter-example:



(13)



$C(G)$



$C(H) = K_9$

The closure of H is a clique.

If $C(H)$ is Hamiltonian $\Rightarrow$ H is Hamiltonian

As $C(G)$ is not Hamiltonian $\Rightarrow$ G is not Hamiltonian

(14) T is a tree $\Rightarrow$ bipartite $\Rightarrow$ $\boxed{\chi(T) = 2}$

Since bipartite $\boxed{\chi'(T) = \Delta(T) = 6}$

$\chi(G,k) = k(k-1)^{n-1}$ for a tree

$$\boxed{\chi(T,k) = k(k-1)^{112}}$$

(15) Assume we have some optimal coloring C with colorset $\{1, 2, \ldots \chi(G)\}$

In C, consider $A = \{\forall v \in V(G) : C(v) = 1\}$

$B = \{\forall u \in V(G) : C(u) = 2\}$

$\vdots$

$X = \{\forall w \in V(G) : C(w) = \chi(G)\}$

Simply run the greedy coloring algorithm with an order given by $\{A, B, \ldots, X\}$

– If we run greedy coloring in order of colors given by some optimal coloring, we'll end up with an optmal coloring

Note: this in no way helps us find such an ordering, however